A Numerical Method for Modelling Discontinuous Mechanics of Asphalt Mixture

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***Abstract*—In order to simulate discontinuous mechanics for asphalt mixture pavement, a new numerical scheme—Meshfree Manifold Method is deduced in this paper by integrating Numerical Manifold Method and Mesh-free Method, which is not only appropriate for contact computation but easy to eliminate the limitation of regular mesh. Some kernel principles are discussedÿand an example is given to prove its efficiency in simulating discontinuous deformation of asphalt mixture.**

***Index Terms*—asphalt mixture, discontinuous mechanics, Numerical Manifold Method, Meshless Method**

I. Introduction

The most used methods to numerate discontinuous mechanics mainly include DEM (Discrete Element Method), DDA (Discontinuous Displacement Analysis) and NMM (Numerical Manifold Method). At present, what successfully applies to asphalt mixture is still DEM, which is accepted as a mature approach to solve the discontinuous problems and large deformation problems. However, DEM bases only on combination of elastic or viscous components to reflect the contact state between particles, therefore, it faces with some difficulties in achieving the stress distribution of asphalt mastic. This paper will introduce NMM into modelling the discontinuous deformation for asphalt mixture.

NMM is established by Genhua Shi in 1991 by using finite covering system in modern manifold [1], which takes the advantages of finite element method and discontinuum kinematics and combines discontinuous and continuous deformation into a uniform mathematical expression, which includes FEM and DDA. Later, Lin [2], Hideomi Ohtsubo [3] and Lu [4] also made a great contribution to the development of NMM. It is not denied that NMM is not universal.

Because of complex structure of asphalt mixture particles, usually FEM-mesh-based NMM faces some difficulties such as volume integration, displacement coordination between materials, and mesh deformity caused by large deformation. Here, a combined method is achieved, in which NMM coverage is replaced by support domain in Meshfree/ Meshless Method (MFM) to deal with asphalt mixture with a large number of gaps, cracks and irregular aggregates and asphalt and other materials. MFM was developed in 1970's, which mainly includes: SPH(Smooth Particle Hydrodynamic Method) [5], RKPM (Reproducing Kernel Particle Method) [6], DEM(Diffuse Element Method) [7], EFGM(the Element-free Galerkin Method) [8] and so on. In theory, Meshfree Manifold Method (MMM) can be very easy to deal with most of discontinuous problems of asphalt mixture, especially for large deformation, cracks, and material coupling calculation.

II. Generally Manifold Covering System

In Numerical Manifold Method, there are two sets of mesh—physical mesh and mathematical mesh defined. Physical meshes are enclosed by objects' borders, cracks, blocks and the interface between different materials, which define the physical region of integration. Mathematical meshes are also named mathematical covers, which only define the precision of approximate solution. They can be any shapes but must cover the entire physical region.

Mathematical covers are divided by physical meshes into physical covers, and the overlap of physical covers is defined as the element of MANIFOLD. It is the basic unit of NMM, and its shape can be arbitrary.

III. Variation Principle Of Minimum Potential Energy

A. Approximate displacement functions

Displacement function is independently defined for each physical cover, and local displacement functions can be connected together to form global displacement function on the entire material domain. If $(u_i(x,y,z), v_i(x,y,z), w_i(x,y,z))$ is the displacement function defined on Physical cover U_i, which can be expressed into a series form as,

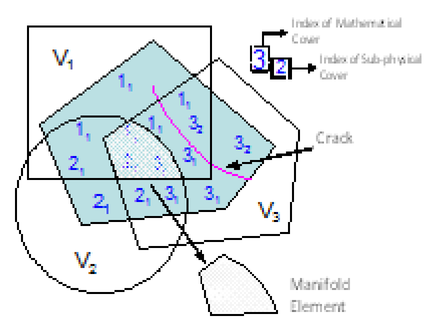

Figure 1. 2D Manifold covering system

$$u_i = \begin{Bmatrix} u_i(x,y,z) \\ v_i(x,y,z) \\ w_i(x,y,z) \end{Bmatrix} = S \cdot D_i \tag{1}$$

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Where, S is the basic series, D_i is the degree of freedom of Physical Cover U_i.

B. Total displacement function

Assuming a point (x, y, z) is covered by q physical covers, the displacement function can be gained by weighted mean of the q displacement approximation functions as follow,

$$\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} = \sum_{i=1}^{q} N_i(x,y,z) \begin{Bmatrix} u_i(x,y,z) \\ v_i(x,y,z) \\ w_i(x,y,z) \end{Bmatrix}$$

$$= \sum_{i=1}^{q} N_i(x,y,z) \cdot S \cdot D_i = T \cdot D \qquad (2)$$

Where, $N_i(x,y,z)$ is weight function of physical cover.

C. The control equations

Assuming there are n physical covers in the solution domain, the system's total potential energy can be expressed as,

$$\Pi = \sum_{j=1}^{n} \Pi_j = \sum_{j=1}^{n} \left(\Pi_e + \Pi_\sigma + \Pi_p + \Pi_w + \Pi_i + \Pi_f \right) \qquad (3)$$

where, Π_e is the strain energy, Π_σ is the initial stress potential energy, Π_p is the potential energy caused by point load, Π_w is the body potential energy, Π_i is the inertial energy, Π_f is the potential energy generated by boundary conditions.

Then, substitute Eq. (2) into Eq. (3), and according to the principle of minimum potential energy, we have the global control equations as

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & \cdots & K_{1n} \\ K_{21} & K_{22} & K_{23} & \cdots & K_{2n} \\ K_{31} & K_{32} & K_{33} & \cdots & K_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ K_{n1} & K_{n2} & K_{n3} & \cdots & K_{nn} \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_n \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{Bmatrix} \qquad (4)$$

Where K_{ii} is only decided by the material of physical cover U_i, and K_{ij} $(i \neq j)$ is the impact matrix of cover U_j on cover U_i.

IV. Combination With Meshless Method

Here, EFG (the Element-free Galerkin Method) is suggested to be used, not only because the coefficient matrix established by this method is symmetrical, but also when there are corresponding fonctionelle for the differential equations, Galerkin method and variational method often lead to the same result.

A. Approximation on Compactly Supported Domain

MLS (*Moving Least Squares*) is known put forward by Lanscaster and Salkamkas in 1982, by which functions $u(x)$ can be expressed approximately by,

$$u(x) \approx u^h(x) = p^T(x) a(x) = \sum_{j=1}^{m} p_j(x) a_j(x) \quad \forall x \in \Omega_x \qquad (5)$$

And, if n points lie in the neighborhood of evaluation point, the weighted squared error norm of $u^h(x)$ in those points is as follow:

$$J = \sum_{i=1}^{n} w_i(x - x_i) \left[p^T(x_i) a_i(x) - u(x_i) \right]^2 \qquad (6)$$

Where, $p^T(x) = \left[p_1(x) \quad p_2(x) \quad \cdots \quad p_m(x) \right]$ is m- times complete basis monomial, $a(x)$ is the coefficient vector, m is that the number of basis monomial, and $w_i(x) = w(x - x_i)$ is the weight function of point x_i in point x.

B. Shape function of Meshfree Manifold Method

Let Eq. (6) obtain the minimum-style, that is,

$$\frac{\partial J}{\partial a} = A(x) \cdot a(x) - B(x) u = 0 \qquad (7)$$

And shape function is gotten:

$$N(x) = p^T(x) A^{-1}(x) B(x) \qquad (8)$$

Where, $A(x) = \sum_{i=1}^{n} w_i(x) p(x_i) p^T(x_i)$

$$B(x) = \left[w_1(x) p(x_1) \quad w_2(x) p(x_2) \quad \cdots \quad w_n(x) p(x_n) \right]$$

At last, substitute Eq. (8) into Eq. (2) to (4), we can obtain the equations of MMM scheme.

V. Numerical Examples

In order to test the validity of MMM, here, a simple asphalt mixture model composed by 16 polyhedral particles is used to analyze the influence of aggregate- asphalt modulus ratio on stress distribution (Fig.2a). The appearance size of this model is 42×42×42(mm), and the diameters of particles subject to gradation composition of AC16. In order to simplify the numeration, the model is only consist of two kinds of particles with diameter of 13.2~16mm and 16~19mm, of which the number of 13.2~16mm aggregate is 11, and of 16~19mm aggregate is 5. Outside size of Asphalt mastic is 1.8 times of aggregate's. Bottom of the model is fixed but the side is free. A vertical pressure is imposed on the top, whose strength varies linearly with the time (p_{min}=0 at 0s, and p_{max}=10N at 1.0s)In addition, aggregate modulus is always assumed with Es = 40000Mpa, and Poisson's ratio is 0.25. For comparison, three different working conditions are selected for asphalt mastic: (i) modulus Ea = 130MPa, Poisson's ratio of 0.3. (ii) modulus Ea = 650MPa, Poisson's ratio of 0.3. (iii) modulus Ea

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= 1300MPa, Poisson's ratio of 0.3. The computation results are shown as Fig.2b) ~d). As shown in Fig.2, the discontinuous features of asphalt mixture model lead to irregular distribution of inner stress, and it is obvious that stress is more concentrated on aggregate. That is to say, stress is always undertaken mostly by the part with larger stiffness.

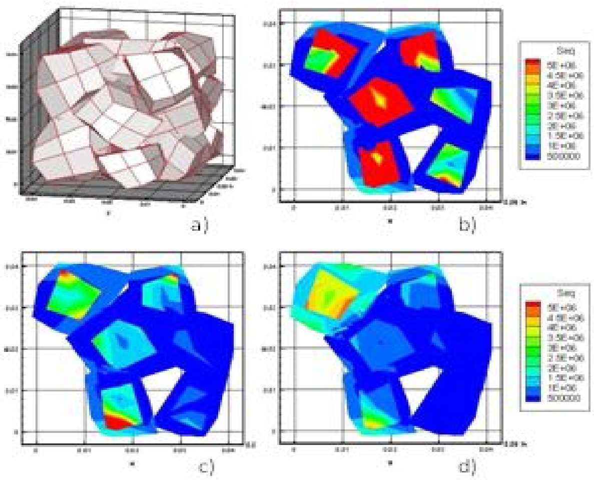

Figure 2. Numerical model (a) and contour of equivalent stress slice when t=0.1s (Es=40Gpa for all, but Ea: b-130Mpa, c-650Mpa, and d-1300Mpa).

Coordination of displacement between aggregate and asphalt mastic caused this phenomenon. If we take the middle particle (3#) in Fig.2a for an example, a data table is easily composed for each working condition (Tab.1). Taking the mean stress ratio of these two materials as evaluation index, we can found that the distribution of asphalt mixture stress is closely related with their modulus ratio: with the ratio increasing, stress in asphalt mastic increases slightly, while stress in aggregate declines dramatically. This shows that: the greater the modulus ratio is, the more stress the aggregate is concentrated; smaller modulus ratio can help stress distributes more evenly.

This conclusion is useful in practice. For example, when the temperature is high (in Summer), the modulus ratio is usually high and stone will bear larger external force. In this condition, to increase the stiffness of asphalt mastic is one of important choices to ensure the mixture to coordinate the distribution of inner stress, such as by adding fiber, using low-grade asphalt or other measures. On the contrary, when the temperature is low (in Winter), lower asphalt modulus is required to reduce the tensile strength by using SBS modified asphalt, etc.. Therefore, in order to give full play to ultimate strength of aggregate and asphalt, we should keep the modulus ratio at a reasonable level to achieve stress coordination between aggregate and asphalt. Additionally, although reducing the aggregate-asphalt modulus ratio can help the inner stress distribution more uniform, while, average stress among different particles differs more largely (Fig.2b~d). Generally, the region where there are more aggregates will have a higher average modulus, while where there is less aggregates will have a lower one. So, it can be said that re- distribution of stress is the integrated result caused by the strength (or stiffness) balance from particles inside and outside.

Table I. Relation Of Mean Equivalent Stress And Modular Ratio For No.3 Granule

Aggregate-asphalt modulus ratio E_s/E_a		40GPa /130MPa	40GPa /650MPa	40GPa /1300MPa
Equivalent stress of aggregate /MPa	Max	17.9151	3.8091	1.8774
	Min	0.3286	0.2720	0.2604
	Mean $\bar{\sigma}_s$	7.6104	1.5779	0.8231
Equivalent stress of asphalt mastic /MPa	Max	0.8664	1.6105	3.0732
	Min	0.0845	0.0477	0.0776
	Mean $\bar{\sigma}_a$	0.2766	0.3239	0.4002
Stress ratio	$\bar{\sigma}_s / \bar{\sigma}_a$	27.51	4.87	2.06

Conclusion

a) Based on the combination of Numerical Manifold Method with Meshfree Method, a efficient numerical method—Meshfree Manifold Method was deduced, which provides a good way to modelling discontinuous mechanics of asphalt mixture. And it is expected useful in simulate the process of crack or large deformation.

Also, the given numerical example shows that:

b) The stress distribution of asphalt mixture is closely related to the modulus ratio of aggregate and asphalt mastic. Although reducing modules ratio can help stress distribution balance inside the particles, but unbalance between the particles.

c) In order to take advantage of ultimate strength of material, it is necessary to keep the mechanical status of aggregate and asphalt mastic harmony by adjusting modulus ratio.

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